Bloom-Gilman duality in the resonance spin structure functions

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We investigate the relations between the spin structure functions in the scaling and resonance regions. We examine the possible duality between the two, and draw inferences for the behavior of the asymmetry A_1 at large x. Finally, we point out the importance of additional polarized structure function data in the resonance region for observing lingering effects of single quark interactions in a region where final state interactions are large. [S0556-2821(98)03521-8]

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I. INTRODUCTION

Duality, in the sense of the Bloom-Gilman duality [1], is a relation between the deep inelastic scattering region and the resonance region in lepton hadron scattering. It states that the smooth scaling curve seen at high momentum transfer is an accurate average over the resonance bumps seen at lower momentum transfer, but at the same value of the Bjorken scaling variable x. The Bloom-Gilman duality is a manifestation of the fact that the single quark reaction rate determines the scale of the reaction rate for the entire process down to remarkably low energies and momentum transfers. In the resonance region, final state interactions are crucial and all quarks must be involved in the reaction. Nevertheless, the overall reaction rate is still determined by the single quark reaction rate—provided we average over regions comparable to the widths of the resonances. So far, these inclusive or exclusive connections have been seen to work for the measured [2,3] unpolarized structure functions [1,4]. Precious little experimental information is available for the role of duality for the polarized structure functions, although efforts in this direction are beginning to bear fruit [5].

Here we investigate the relevance and consequences of duality for the polarized structure functions. We shall show that perturbative QCD (PQCD) arguments lead to an inclusive or exclusive relationship in the polarized case akin to that in the unpolarized one. Also, we shall discuss perturbative expectations for resonance contributions in the high x limit of the polarization asymmetry. Finally, we note interesting features of the present data and point out what can be learned from additional polarized structure function data in the resonance region at higher Q^2.

II. DEFINITIONS, KINEMATICS, AND RELATIONS

We begin with some definitions and kinematic relations. For deep inelastic scattering, $e+p\to e+X$, the structure functions W_i and G_i are defined by

$$\begin{aligned} W_{\mu\nu}=&\frac{1}{4\pi m_N}\sum_X (2\pi)^4\delta^4(q+p-p_X)\\ &\times\langle ps|j_\mu(0)|X\rangle\langle X|j_\nu(0)|ps\rangle\\ =&-g_{\mu\nu}W_1+\frac{1}{m_N^2}p_\mu p_\nu W_2+\frac{i}{m_N}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\\ &\times\left[s^\sigma G_1+\frac{1}{m_N^2}(p\cdot q s^\sigma - s\cdot q p^\sigma)G_2\right], \end{aligned} \tag{1}$$

where s is the spin vector of the nucleon target and satisfies $s\cdot p=0$ and $s^2=-1$, p is the momentum of the target nucleon, and q is the momentum of the incoming virtual photon. Often, the structure functions are replaced by

$$\nu W_2=F_2,\quad m_N W_1=F_1,$$

$$\nu G_1=g_1,\quad \frac{\nu^2}{m_N}G_2=g_2 \tag{2}$$

where it is expected that F_i and g_i depend just on $x\equiv Q^2/2m_N\nu$ (where $Q^2=-q^2$) in the scaling region, up to logarithmic corrections.

The exclusive process $e+p\to e+R$, where R stands for the final baryon, a resonance or the nucleon (in the case of elastic scattering), is described by the helicity amplitudes

$$G_m=\frac{1}{2m_N}\langle R,\lambda'=m-\tfrac{1}{2}|\epsilon_\mu^{(m)}\cdot j^\mu(0)|N,\lambda=\tfrac{1}{2}\rangle. \tag{3}$$

The photon polarization vectors are

$$\epsilon^{(\pm)}=(0,\mp1,-i,0)/\sqrt{2},$$

$$\epsilon^{(0)}=\frac{1}{Q}(|\vec{q}|,0,0,\nu), \tag{4}$$

with $q=(\nu,0,0,|\vec{q}|)$.

Note that the helicity of the final baryon is

$$\begin{aligned} &\frac{1}{2} \quad \text{for } G_+ , \\ -&\frac{1}{2} \quad \text{for } G_0 , \\ -&\frac{3}{2} \quad \text{for } G_- . \end{aligned} \tag{5}$$

Thus if the final baryon has spin 1/2, G_- must be absent. For elastic scattering, the helicity amplitudes are related to well-known form factors by

$$G_+ = \frac{Q}{m_N\sqrt{2}} G_M ,$$
$$G_0 = G_E . \tag{6}$$

For the non-elastic case, one often uses the amplitudes

$$|A_{1/2,3/2}| = e\sqrt{\frac{m_N}{m_R^2 - m_N^2}}\, |G_{+,-}|$$

where e is the proton charge.

For a single sharp resonance R the relations between the structure functions and the helicity amplitudes are

$$F_1 = m_N^2 \delta(W^2 - m_R^2)[|G_+|^2 + |G_-|^2],$$
$$\left(1+\frac{\nu^2}{Q^2}\right)F_2 = m_N \nu\, \delta(W^2 - m_R^2) \times [|G_+|^2 + 2|G_0|^2 + |G_-|^2],$$
$$\left(1+\frac{Q^2}{\nu^2}\right)g_1 = m_N^2 \delta(W^2 - m_R^2) \times \left[\left|G_+\right|^2 - \left|G_-\right|^2 + (-1)^{s_R - 1/2}\eta_R \frac{Q\sqrt{2}}{\nu} G_0^* G_+\right],$$
$$\left(1+\frac{Q^2}{\nu^2}\right)g_2 = -m_N^2 \delta(W^2 - m_R^2) \times \left[\left|G_+\right|^2 - \left|G_-\right|^2 - (-1)^{s_R - 1/2}\eta_R \frac{\nu\sqrt{2}}{Q} G_0^* G_+\right], \tag{7}$$

where $W^2 \equiv (p+q)^2$, the total hadronic mass squared, and s_R and η_R are the spin and parity of the resonance. The delta function for the sharp resonance can be most simply approximated by

$$\delta(W^2 - m_R^2) \approx \frac{1}{2m_R} \frac{\Gamma_R/2\pi}{(W-m_R)^2 + \Gamma_R^2/4} \xrightarrow{peak} \frac{1}{\pi m_R \Gamma_R}, \tag{8}$$

with Γ_R being the width of the resonance.

III. SCALING PROPERTIES OF $G_{\pm,0}$ AND $G_{1,2}$

Let us now discuss the scaling properties of $G_{\pm,0}$ and $g_{1,2}$. The resonance contributions to the structure functions fall with increasing Q^2 and also move to progressively higher x, approaching $x=1$. They thus may be described as falling with x, at a certain rate. We wish to determine if the falloff rate is the same as that in the deep inelastic region as $x \to 1$, but at much higher Q^2. This is already known to be true for the spin independent structure functions, and the phenomenon is known as Bloom-Gilman duality [1]. Hence, we will concentrate our attention on the spin dependent structure functions.

The counting rules [6] give the following behavior at high Q^2 for the helicity amplitudes (modulo logarithms) [4]:

$$G_+ = g_+/Q^3, \quad G_0 = (m_N) g_0/Q^4,$$
$$G_- = (m_N)^2 g_-/Q^5, \tag{9}$$

where $g_{\pm,0}$ are constants, real in leading Born order, and the mass factors are put in purely for dimensional reasons. This allows us to find the behavior due to the resonance of g_1 at the resonance peak and at high Q^2 to be

$$g_1 = \frac{m_N^2}{\pi m_R \Gamma_R} \frac{g_+^2}{Q^6} = \frac{m_N^2}{\pi m_R \Gamma_R} \frac{g_+^2}{(m_R^2 - m_N^2)^3}(1-x)^3. \tag{10}$$

The second result requires

$$\frac{1}{Q^2} = \frac{1}{W^2 - m_N^2} \frac{1-x}{x} \approx \frac{1}{m_R^2 - m_N^2}(1-x) \tag{11}$$

for $x \to 1$ and $W \approx m_R$. Similarly,

$$g_2 = -\frac{m_N^2}{\pi m_R \Gamma_R} \frac{(1-x)^3}{(m_R^2 - m_N^2)^3} g_+ \left(g_+ - \frac{\eta_R(-1)^{s_R - 1/2}}{\sqrt{2}} g_0\right), \tag{12}$$

for high Q^2. It is interesting to note that the high Q^2 resonance contributions to the polarized structure functions can be inferred from unpolarized structure function measurements, since g_+ and g_0 are the transverse and longitudinal contributions that are individually obtained from a Rosenbluth separation. If g_2 is small, as seems to be indicated [7], then there must be a relation between the transverse and longitudinal resonance form factors, viz.,

$$\sqrt{2} g_+ \approx \eta_R (-1)^{s_R - 1/2} g_0 . \tag{13}$$

In the deep inelastic region, the spin structure function g_1 is related to the quark distributions in a manner similar to F_1 except for one sign,

$$g_1=\frac{1}{2}\sum e_q^2[q_\uparrow(x,Q^2)-q_\downarrow(x,Q^2)],$$

$$F_1=\frac{1}{2}\sum e_q^2[q_\uparrow(x,Q^2)+q_\downarrow(x,Q^2)]. \tag{14}$$

The $q_{\uparrow,\downarrow}$ are the quark distributions for quark helicities parallel or antiparallel to the parent nucleon polarization. Perturbative QCD dictates that $q_\uparrow$ dominate as $x\to 1$ [8]. If so, the $x\to 1$ behavior will be the same for both functions. Even if PQCD did not work for the $x\to 1$ limit, it would require a remarkable cancellation to make the high x behavior different for F_1 and g_1. Since the behavior of F_1 is well known, we can conclude

$$\lim_{x\to 1} g_1(x)\propto(1-x)^3 \tag{15}$$

(modulo perturbative evolution effects [9]) in the deep inelastic region. This is the same as the contribution from the resonance region. One part of the duality between the resonance and deep inelastic regions is thus established.

It is less clear what the deep inelastic result should be for g_2. There is no unique parton model formula for it. However, accepting the Wandzura-Wilczek relation [10]

$$g_2^{WW}(x)=-g_1(x)+\int_x^1 dx'\,\frac{g_1(x')}{x'}, \tag{16}$$

leads to the result that

$$\lim_{x\to 1} g_2^{WW}(x)=-g_1(x)\propto(1-x)^3. \tag{17}$$

Thus if this is the dominant piece at high x, the scaling part of the duality is established for g_2 also.

IV. REMARKS ON THE BEHAVIOR OF A_1 OR g_1

Now we shall further examine the $x\to 1$ behavior of g_1, or of the photon asymmetry A_1:

$$A_1\equiv\frac{\sigma_{1/2}-\sigma_{3/2}}{\sigma_{1/2}+\sigma_{3/2}}=\frac{g_1-\dfrac{Q^2}{\nu^2}g_2}{F_1}, \tag{18}$$

where the cross sections are for photon absorption with initial state spin projections of 1/2 and 3/2. For a resonance,

$$A_1=\frac{|G_+|^2-|G_-|^2}{|G_+|^2+|G_-|^2}. \tag{19}$$

For the elastic point, $x=1$, there is only the nucleon and $G_-=0$ so that rigorously

$$A_1(x=1)=1. \tag{20}$$

For a single resonance of spin 1/2, the same is true. Even for spin 3/2 and higher resonances, the scaling rules tell us that $|G_+|\gg|G_-|$ at high Q^2, so that $A_1\to 1$ as $x\to 1$. If the only backgrounds under a given resonance are due to tails of other resonances, then the same rule still applies:

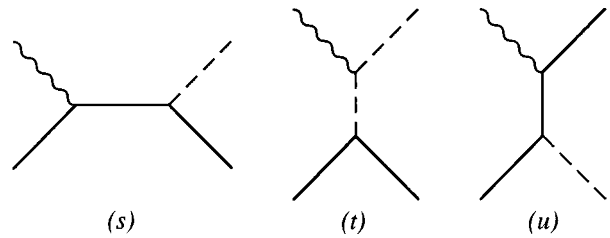

FIG. 1. The s, t, and u channel Born terms for electroproduction of pion-nucleon final states.

$$\lim_{Q^2\to\infty} A_1=1 \quad \text{if only a resonant background}. \tag{21}$$

However, if the non-resonant background is dominated by Born terms [11], we can get $A_1\to 1$ anyway. In the resonance region the t-channel and u-channel diagrams (see Fig. 1) have propagators that suppress their contributions at high Q^2, leaving an s-channel diagram which has only $\sigma_{1/2}$. (Purely as an aside, a dominant t-channel Born term is needed for certain measurements of the pion form factor, and this can indeed happen even at high Q^2, but only if the final hadronic state is well outside the resonance region.) Note that the isospin of the resonance plays no role in the above considerations.

The $x\to 1$ behavior of A_1 in the scaling region can be obtained from the ratio of the two equations (14). We quote the results for PQCD, where the $x\to 1$ results for the polarized quark distributions are mentioned above: for SU(6), where no distinction is made between the distributions of differently polarized quarks and for a number of modern suggestions for the polarized quark distributions [12]. One has

$$\lim_{x\to 1} A_1=\begin{cases} 1 & \text{PQCD or Soffer and co-workers [12]},\\ \dfrac{5}{9} & \text{SU(6)},\\ 0.75 & \text{GS, version B [12]}\\ 0.66 & \text{GRSV (``standard'' NLO) [12]}\end{cases}. \tag{22}$$

The last two are given at their respective benchmark Q^2.

V. PRESENT DATA AND CONCLUSIONS

Let us look at the present relevant data on the polarized structure functions in the resonance region. There is a paucity of such data. The SLAC measurements, from the E143 Collaboration recently [5] (and from earlier data with larger error bars [13]), do cover $W^2<5\,\mathrm{GeV}^2$ at $Q^2\approx 0.5$ and 1.2 GeV^2. These Q^2 are still too low for a duality test, since duality is not working at these Q^2 in the spin-independent case [1,4]. Nevertheless, it is useful to discuss the data that exist.

In the first resonance region, the $\Delta(1232)$ itself gives nearly all the signal at very low Q^2 in the unpolarized case and may be expected to do the same in the polarized case.

Further, at low Q^2, Δ electroproduction is dominated by the magnetic dipole amplitude [14,15,16], which leads to $|G_-| \approx\sqrt{3}|G_+|$ and

$$A_1(\Delta,\text{low } Q^2)\approx -1/2. \tag{23}$$

Abe *et al.* [7] find for $Q^2\approx 0.5$ GeV^2 and in the Δ region that $A_1\approx -1/3$, in qualitative agreement with our expectation. However, for $Q^2\approx 1.2$ GeV^2, the measured value of A_1 is consistent with zero (albeit also consistent with $-1/2$ at a 2σ level). Since there is evidence that the M1 dominance is still valid for the resonance itself, the A_1 result must be due to the background and resonance giving approximately canceling contributions. This suggests a violation of the strict construction of Bloom-Gilman duality, since the Q^2 dependences of the resonance and background do not match. However, Q^2 is still low.

It is reminiscent of the unpolarized case, where for the Δ Bloom-Gilman duality works (above a few GeV^2) in the sense of the resonance region average matching the scaling curve, and does so without having the resonance to continuum ratio be constant, but rather because of an interplay between resonance and continuum [4]. As one falls, the other rises, relative to the scaling curve, and the sum stays about the same. So as in the unpolarized case, the sum over channels allows matching the scaling curve at low Q^2, perhaps so in the polarized case the sum over channels will show the perturbative polarization prediction at a lower Q^2 when a single channel will not.

For the second resonance region, the prediction for A_1 involves the $S_{11}(1535)$ and $D_{13}(1520)$, as well as the nonresonant background. At very low Q^2, the largest resonant amplitude is the $A_{3/2}$ for exciting the D_{13} [17], the next largest is the $A_{1/2}$ for the S_{11} [18], and $A_{1/2}$ for the D_{13} is quite small. However, the resonances soon reconcile themselves to the high Q^2 expectations; as for the D_{13}, the $A_{1/2}$ and $A_{3/2}$ change relative size in the vicinity of 1 GeV^2 [19]. Hence, considering the resonant contributions alone, we expect A_1 to be negative at low Q^2 and become positive before 1 GeV^2. The available data [5] show A_1 to be positive at both 0.5 and 1.2 GeV^2.

For polarized structure functions, in contrast to the unpolarized case, duality must break down spectacularly at low enough Q^2 [20]. The argument goes by considering the Ellis-Jaffe integral [21], written as

$$\int_{\nu_0}^{\infty}\frac{d\nu}{\nu^2}\,g_1^p=\frac{2m_N\Gamma^p}{Q^2}, \tag{24}$$

with Γ^p approximately constant at high Q^2, and comparing it to the Drell-Hearn-Gerasimov sum rule [22], which may be written in the form

$$\int_{\nu_0}^{\infty}\frac{d\nu}{\nu^2}\,g_1^p(\nu,Q^2=0)=-\frac{\kappa_p^2}{2m_N}. \tag{25}$$

Above, κ_p is the anomalous magnetic moment of the proton, and ν_0 is given by the pion production threshold. The quantity Γ_p is measured to be positive at Q^2 of several GeV^2 (see, for example, [5]). If g_1 were on the average the same at very low Q^2 as it is at high Q^2, then the right hand side of the Drell-Hearn-Gerasimov sum rule would be positive—and it is clearly not.

We have shown that polarization structure function data in the resonance region at higher Q^2 are interesting and can throw significant new light on the issue of duality. Facilities such as Jefferson Lab, SLAC, and HERMES can all contribute over a significant range of Q^2 and W. The idea that the single quark cross section sets the scale on the average even in the resonance region gets a new field of exploration in the polarized structure function. Unlike the unpolarized case, one expects its breakdown at sufficiently low Q^2. Finding this breakdown will signal the onset of a region where the final state interactions obliterate even a remnant of perturbative physics.

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[1] E. D. Bloom and F. J. Gilman, Phys. Rev. Lett. **25**, 1140 (1970); Phys. Rev. D **4**, 2901 (1971).

[2] P. Stoler, Phys. Rev. Lett. **66**, 1003 (1991); Phys. Rev. D **44**, 73 (1991); Phys. Rep. **226**, 103 (1993); G. Sterman and P. Stoler, Annu. Rev. Nucl. Part. Sci. **47**, 193 (1997).

[3] P. E. Bosted *et al.*, Phys. Rev. D **49**, 3091 (1994); C. Koeppel, Ph.D. thesis, American University, 1994.

[4] C. E. Carlson and N. C. Mukhopadhyay, Phys. Rev. D **41**, R2343 (1990); **47**, R1737 (1993); Phys. Rev. Lett. **74**, 1288 (1995).

[5] K. Abe *et al.*, Phys. Rev. Lett. **78**, 815 (1997).

[6] S. J. Brodsky and G. Farrar, Phys. Rev. Lett. **31**, 1153 (1973), Phys. Rev. D **11**, 1309 (1975); V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze, Lett. Nuovo Cimento **7**, 719 (1973).

[7] K. Abe *et al.*, Phys. Lett. B **404**, 377 (1997); K. Abe *et al.*, Phys. Rev. Lett. **76**, 587 (1996); D. Adams *et al.*, *ibid.* **336**, 125 (1994); K. Abe *et al.*, Phys. Rev. D (to be published) hep-ph/9802357.

[8] G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **35**, 1416 (1975).

[9] We should note a suggestion that perturbative evolution may be diminished for large x and moderate Q^2. See S. J. Brodsky, Tao Huang, and G. P. Lepage, in *Proceedings of the Banff Summer Institute on Particles and Fields*, Banff, Canada, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum Press,

New York, 1983), especially pp. 177–178.

[10] S. Wandzura and F. Wilczek, Phys. Lett. **72B**, 195 (1977); R. Jaffe, Comments Nucl. Part. Phys. **19**, 239 (1990).

[11] R. M. Davidson, N. C. Mukhopadhyay, and R. S. Wittman, Phys. Rev. D **43**, 71 (1991).

[12] F. Buccella and J. Soffer, Phys. Rev. D **48**, 5416 (1993); C. Bourrely and J. Soffer, *ibid.* **53**, 4067 (1996); T. Gehrmann and W. J. Stirling, *ibid.* **53**, 6100 (1996); M. Glück, E. Reya, M. Stratmann, and W. Vogelsang, *ibid.* **53**, 4775 (1996).

[13] G. Baum *et al.*, Phys. Rev. Lett. **45**, 2000 (1980).

[14] V. D. Burkert and L. Elouadrhiri, Phys. Rev. Lett. **75**, 3614 (1995).

[15] R. M. Davidson and N. C. Mukhopadhyay, Phys. Lett. B **353**, 151 (1995).

[16] C. E. Carlson and J. L. Poor, Phys. Rev. D **38**, 2758 (1988); G. R. Farrar, H. Zhang, A. Oglublin, and I. R. Zhitnitsky, Nucl. Phys. **B311**, 585 (1989); J. Bonekamp, Bonn report, 1989.

[17] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[18] B. Krusche *et al.*, Phys. Lett. B **397**, 171 (1997); N. C. Mukhopadhyay, J.-F. Zhang, and M. Benmerrouche, Phys. Rev. Lett. **75**, 3022 (1995).

[19] V. Burkert, in *Research Program at CEBAF, 1986*, edited by Franz Gross (CEBAF, Newport News, 1986).

[20] M. Anselmino, B. L. Ioffe, and E. Leader, Yad. Fiz. **49**, 214 (1989) [Sov. J. Nucl. Phys. **49**, 136 (1989)]; F. Close, in *Excited Baryons 1988*, Proceedings of the Topical Workshop on Excited Baryons, edited by G. Adams, N. C. Mukhopadhyay, and P. Stoler (World Scientific, Singapore, 1989); V. D. Burkert and B. L. Ioffe, J. Exp. Theor. Phys. **78**, 619 (1994).

[21] J. Ellis and R. Jaffe, Phys. Rev. D **9**, 1444 (1974); **10**, 1669(E) (1974); see also J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); Phys. Rev. D **1**, 1376 (1970).

[22] S. D. Drell and A. C. Hearn, Phys. Rev. Lett. **16**, 908 (1966); S. B. Gerasimov, Sov. J. Nucl. Phys. **2**, 598 (1966).